Enrollment No: _______________________ **Exam Seat No:** _______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Mathematical Methods-II

Subject Code: 5SC04MAM1 **Branch: M.Sc. (Mathematics)**

Semester: 4 **Date: 08/05/2018** **Time:10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions** **(07)**

a. Prove that the shortest distance between two points in a plane is a straight line. (02)

b. Derive the second form of Euler's equation. (02)

c. Find the extremalsof $I[y(x)] = \int_{x_0}^{x_1} (16y^2 - y''^2 + x^2)\, dx$. (02)

d. Define: Integro-Differential equation. (01)

Q-2 **Attempt all questions** **(14)**

a. Prove that if the functional $I[y(x)] = \int_{x_1}^{x_2} f(x, y, y')\, dx$ has the extremum value, then the integrand f satisfies the Euler's equation $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. (06)

b. Find the curve passing through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ which when rotated about the x-axis gives a minimum surface area. (05)

c. Prove that the extremal of the isoperimetric problem $I[y(x)] = \int_1^4 y'^2\, dx$, $y(1) = 3, y(4) = 12$, subject to the condition $\int_1^4 y\, dx = 36$. (03)

OR

Q-2 **Attempt all questions** **(14)**

a. Find the geodesics on a right circular cylinder of radius a. (06)

b. Find the plane curve of fixed perimeter and maximum area. (05)

c. Show that the functional $I[x(t), y(t)] = \int_0^1 (x'^2 + y'^2)\, dt$ with $x(0) = 0$, $y(0) = 1, x(1) = 1.5, y(1) = 2$ is stationary for $x(t) = 1.5t$ and $y(t) = t + 1$. (03)

Q-3 **Attempt all questions** **(14)**

a. State Leibniz's rule. Prove that (08)

$$\int_a^x \int_a^{x_n} \dots \int_a^{x_2} f(x_1)dx_1 dx_2 \dots dx_n = \frac{1}{(n-1)!} \int_a^x (x-t)^{n-1} f(t)\, dt$$

b. Find the extremals of the functional $\int_0^{\pi/2}(y''^2 - y^2 + x^2)dx$that satisfying the conditions $y(0) = 1, y'(0) = 0, y\left(\frac{\pi}{2}\right) = 0, y'\left(\frac{\pi}{2}\right) = -1.$ (06)

OR

Q-3 Attempt all questions (14)

a. Find the integral equation corresponding to the boundary value problem $6y''(x) - 4y'(x) + y(x) = 4\,cos(2x)\ ,\ \ y(0) = -1, y'(0) = 4.$ (08)

b. Find the extremals of the functional $I[x(t), y(t)] = \int_0^1 (x'y' + 2x^2 + 2y^2)\,dt$ with $x(0) = y(0) = 0, x(\pi/4) = \ y(\pi/4) = 1.$ (06)

SECTION – II

Q-4 Attempt the following questions (07)

a. State Fredholm integral theorem. (02)

b. Find the eigenvalue and corresponding eigenfunction of $y(x) = \lambda \int_0^1 e^{x+t} y(t)dt.$ (02)

c. Prove that if $y_1(x)$ and $y_2(x)$ are solutions of the homogeneous Fredhom integral equation $y(x) = \lambda \int_a^b K(x,t)y(t)dt$,then $y(x) = \alpha y_1(x) + \beta y_2(x)$ is also a solution, where α and β are arbitrary constants. (02)

d. Define:Degenerate kernel. (01)

Q-5 Attempt all questions (14)

a. Solve the integro-differential equation$y' = 3\int_0^x \cos 2(x-t)\,y(t)\ dt + 2$,where $y(0) = 1.$ (07)

b. Show that $y(x) = xe^x$ is a solution of $y(x) = \sin x + 2\int_0^x \cos(x-t)y(t)dt.$ (04)

c. State Chebyshev differential equation and reduce it to Strum-Liouville equation. (03)

OR

Q-5 Attempt all questions (14)

a. Discuss the eigenvalues and corresponding eigenfunctions for the integral equation $y(x) = F(x) + \lambda \int_0^1 (1-3xt)y(t)dt$ (07)

b. Solve the Abel's integral equation$\int_0^x \frac{y(t)}{(x-t)^{\frac{1}{3}}}\,dt = x(x+1).$ (04)

c. Solve the integral equations $y(x) = 3\sin x + \int_0^x (x-t)y(t)dt.$ (03)

Q-6 Attempt all questions (14)

a. Find the eigenvalues and the corresponding eigenfunctions of the differential equation $y'' + \lambda\, y = 0$ on the interval $[0, \pi]$ with the boundary conditions $y'(0) = 0$and $y(\pi) = 0.$ (07)

b. Reduce the differential equation$xy'' + \alpha y' + k^2xy = 0$ into Bessel's differential equation taking $y = x^n z.$ (07)

OR

Q-6 Attempt all questions (14)

a. Reduce the differential equation $xy'' + 5y' + xy = 0$into Bessel's differential equation taking$x^2y = z$and reduce it to Sturm-Liouvilledifferential equation. (07)

b. Solve the integral equation $y(x) = x + \lambda \int_0^1 (1 + x + t)y(t)dt.$ (07)